Контрольная работа № 9

ТЕОРИЯ ФУНКЦИЙ КОМПЛЕКСНОГО ПЕРЕМЕННОГО. ОПЕРАЦИОННОЕ ИСЧИСЛЕНИЕ. ИНТЕГРАЛ ФУРЬЕ

Задача 1[13]. Пользуясь условиями Коши-Римана, установить дифференцируемость функции $f(z)$ и найти $f'(z_0)$.

1.1.	$f(z) = z^2 + e^z,$	$z_0 = 0.$	1.2.	$f(z) = e^z,$	$z_0 = 0.$
1.3.	$f(z) = z + \sin z,$	$z_0 = 0.$	1.4.	$f(z) = \sin z,$	$z_0 = 0.$
1.5.	$f(z) = z^{-1},$	$z_0 = 1.$	1.6.	$f(z) = z^2 e^z,$	$z_0 = 0.$
1.7.	$f(z) = \dfrac{z^3 - 2iz + i}{z},$	$z_0 = i.$	1.8.	$f(z) = z^2 + 2z,$	$z_0 = i.$
1.9.	$f(z) = \dfrac{i}{z} + iz,$	$z_0 = 1.$	1.10.	$f(z) = ze^{iz},$	$z_0 = 0.$
1.11.	$f(z) = z \sin z,$	$z_0 = -1.$	1.12.	$f(z) = \dfrac{z-1}{z+1},$	$z_0 = i.$
1.13.	$f(z) = \dfrac{1}{z} \ln z,$	$z_0 = i.$	1.14.	$f(z) = e^z,$	$z_0 = i.$
1.15.	$f(z) = z \cos z,$	$z_0 = 0.$	1.16.	$f(z) = \dfrac{z^2(3+2i)}{2(z-1)},$	$z_0 = 0.$
1.17.	$f(z) = z^4,$	$z_0 = i.$	1.18.	$f(z) = z \cos z + z,$	$z_0 = 0.$
1.19.	$f(z) = \dfrac{1}{z}(\sin z + 1),$	$z_0 = 1.$	1.20.	$f(z) = z + \dfrac{1}{z},$	$z_0 = 1.$
1.21.	$f(z) = z^2 \sin z,$	$z_0 = 0.$	1.22.	$f(z) = \dfrac{\cos z}{z},$	$z_0 = 1.$
1.23.	$f(z) = \dfrac{1}{z^2} + \dfrac{1}{z},$	$z_0 = 1.$	1.24.	$f(z) = \dfrac{z^2-1}{z^2+1},$	$z_0 = -2i.$
1.25.	$f(z) = \operatorname{tg} z,$	$z_0 = -i.$	1.26.	$f(z) = \sin z + \cos z,$	$z_0 = \pi.$
1.27.	$f(z) = \dfrac{1}{\sin z},$	$z_0 = \dfrac{\pi}{2}.$	1.28.	$f(z) = \dfrac{\cos z}{\sin^2 z},$	$z_0 = \dfrac{\pi}{2}.$
1.29.	$f(z) = \dfrac{z}{\operatorname{tg} z},$	$z_0 = \pi.$	1.30.	$f(z) = \dfrac{\operatorname{tg} z}{z},$	$z_0 = i.$

Задача 2[14]. Вычислить интеграл, используя вычеты.

2.1. $$\oint\limits_{|z+2i|=3} \left(e^{1/(z+i)} + \frac{7}{z(z^2+4)} \right) dz.$$

[13] Автор – Н.В.Полухин. Для образца см. решение задач 3.8-3.13, а также задачи 3.2 в Части 4, т.1.

[14] Автор – В.И.Голинько. Для образца см. решение задач 3.37-3.38, а также задач 3.32-3.36 в Части 4, т.1.

2.2. $$\oint\limits_{|z|=\frac{3}{2}} \left(\sin\frac{2}{z} + \frac{z-2}{(z-i)^2(z^2+z-2)}\right) dz.$$

2.3. $$\oint\limits_{|z+i|=2} \left(\cos\left(1+\frac{1}{z}\right) + \frac{z}{(z^2-z-2)^2} + \frac{e^z}{z+i}\right) dz.$$

2.4. $$\oint\limits_{|z|=2} \left(z^2\sin\frac{1}{z^3} + \frac{z+4}{z^2(z^2+2z-3)}\right) dz.$$

2.5. $$\oint\limits_{|z-i|=2} \left(e^{1/(z-1)} + \frac{\sin z}{z^3(z^2+4)}\right) dz.$$

2.6. $$\oint\limits_{|z-\pi|=3} \left(\operatorname{sh}\frac{1}{z-1} + \frac{z+3}{(z^2+4z)^2} + \frac{z}{1+\cos z}\right) dz.$$

2.7. $$\oint\limits_{|z-1-i|=\frac{3}{2}} \left(z\operatorname{ch}\frac{1}{z-2} + \frac{2}{z(z^2-4)^2}\right) dz.$$

2.8. $$\oint\limits_{|z|=3} \left(\frac{2}{z+i}\cos\frac{1}{z+i} + \frac{z+i\pi}{e^z+1} + \frac{z}{(z^2+6z+8)^2}\right) dz.$$

2.9. $$\oint\limits_{|z+2|=2} \left((z+3)\sin\frac{1}{z-1} + \frac{z-4}{(z+1+i)(z^2-z)^2}\right) dz.$$

2.10. $$\oint\limits_{|z+1+i|=\frac{3}{2}} \left(z\operatorname{sh}\frac{1}{z-i} + \frac{\sin z}{z^2(z^2-4)^2}\right) dz.$$

2.11. $$\oint\limits_{|z-1|=2} \left(z\operatorname{ch}\frac{1}{z-1-i} + \frac{z+3}{(z^2-4)(1-e^z)^2}\right) dz.$$

2.12. $$\oint\limits_{|z|=2} \left(z\sin\left(1+\frac{1}{z}\right) + \frac{4}{z(z^2-2z-3)^2}\right) dz.$$

2.13. $$\oint\limits_{|z-1|=3} \left(e^{i/(z-1-i)} + \frac{z}{\sin^2 z} + \frac{2z+3}{z^2+25}\right) dz.$$

2.14. $$\oint\limits_{|z+2i|=3} \left(z\cos\frac{1}{z} + \frac{z-2}{(z-i+1)^2(z^2+3z-4)}\right) dz.$$

2.15. $$\oint\limits_{|z-2|=3} \left(e^{1-\frac{1}{z+i}} + \frac{z}{(\cos z-1)^2}\right) dz.$$

2.16. $$\oint\limits_{|z-i|=3} \left(\cos\left(1-\frac{1}{z}\right) + \frac{z+2}{(z+i)^2(z^2-2z-3)}\right) dz.$$

2.17. $$\oint\limits_{|z+i|=2} \left((z+2)e^{1/z} + \frac{\cos 2z}{(z+1)^2(z^2+4)}\right) dz.$$

2.18. $$\oint\limits_{|z+i|=\frac{3}{2}} \left(\sin\left(2-\frac{1}{z+1}\right) + \frac{2}{z^2(z^2+z-2)}\right) dz.$$

2.19. $$\oint\limits_{|z+2|=\frac{3}{2}} \left(ze^{3/(z+2)} + \frac{4}{(z+1)^2(4z^2-9)}\right) dz.$$

2.20. $$\oint\limits_{|z-1|=2} \left(\operatorname{sh}\left(1+\frac{1}{z-2}\right) + \frac{e^z+1}{(z+i)^2(z^2-4z)}\right) dz.$$

2.21. $$\oint\limits_{|z-3i|=5} \left(\operatorname{ch}\left(2+\frac{i}{z-i}\right) + \frac{\cos^2 z}{z^2(z^2+3z)-10}\right) dz.$$

2.22. $$\oint\limits_{|z+5i|=7} \left(\frac{3}{z} e^{1/z^2} + \frac{\sin z}{(z^2-8z+15)(z-i)^2} \right) dz.$$

2.23. $$\oint\limits_{|z-i|=3} \left(\sin\frac{1}{z+i} + \frac{z+2}{(e^z-1)^2(z^2+z-12)} \right) dz.$$

2.24. $$\oint\limits_{|z+1|=3} \left(\cos\frac{1}{z-2i} + \frac{e^{iz}}{z(z^2-2z-8)^2} \right) dz.$$

2.25. $$\oint\limits_{|z+i|=3} \left(\frac{2}{z}\,\mathrm{ch}\,\frac{1}{z} + \frac{2z+3}{(z^2+9)^2(z^2-4z-5)} \right) dz.$$

2.26. $$\oint\limits_{|z-1|=3} \left(z^2 \sin\frac{3}{z} + \frac{\cos z}{(z^2-9)^2(z-1-i)} \right) dz.$$

2.27. $$\oint\limits_{|z-i\pi|=4} \left(e^{2/(z-i\pi)} + \frac{z+6}{z(z+1+i)\sin z} \right) dz.$$

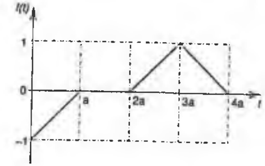
2.28. $$\oint\limits_{|z|=2} \left(\frac{3}{z} + z\cos\frac{1}{z} + \frac{e^z+z}{(z-1)(z^2+2z-3)^2} \right) dz.$$

2.29. $$\oint\limits_{|z-1+i|=2} \left(\mathrm{sh}\,\frac{1}{z+i} + \frac{z-2}{(z^4-16)^2} \right) dz.$$

2.30. $$\oint\limits_{|z-2|=3} \left((z+1)e^{1/z} + \frac{z-\pi}{\sin z(z^2-8z+12)^2} \right) dz.$$

Задача 3[15]. Найти изображение функции, заданной графически.

[15] Автор – Н.В.Полухин. Для образца решения см. пример 4.12 в Части 4, т.1.

3.1.


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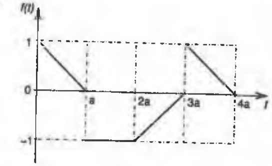

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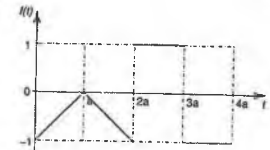

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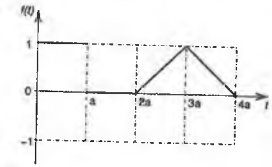

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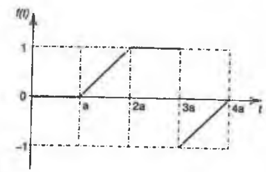

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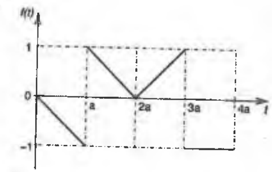

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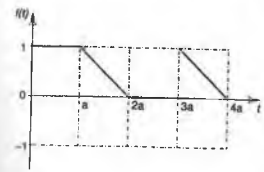

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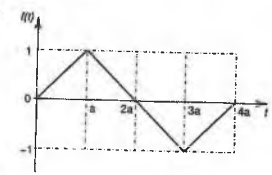

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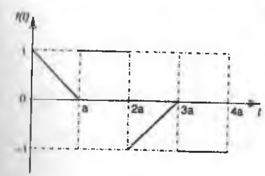

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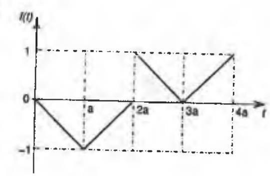

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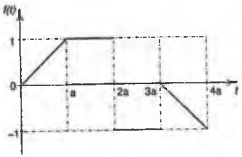

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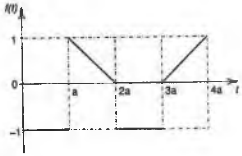

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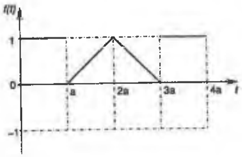

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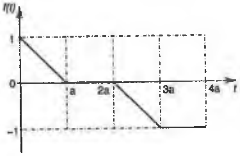

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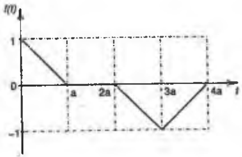

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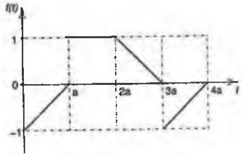

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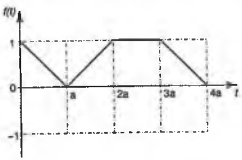

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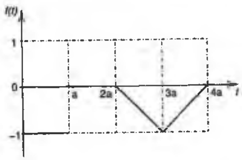

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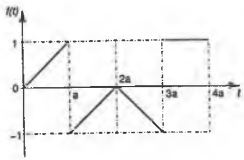

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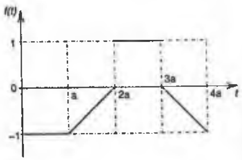

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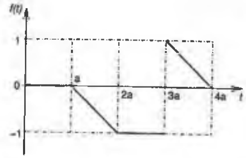

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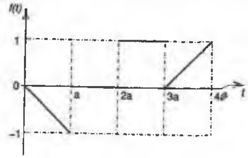

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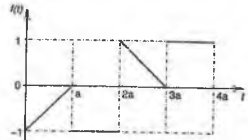

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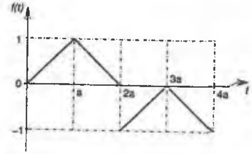

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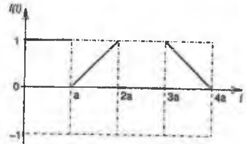

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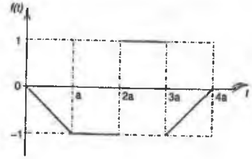

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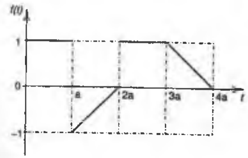

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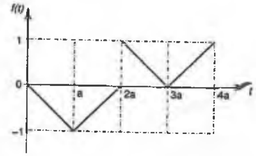

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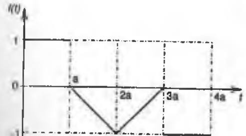

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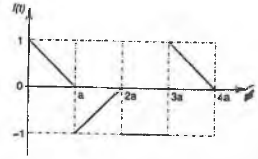

Задача 4[16]. Методом операционного исчисления найти решение задачи Коши.

4.1. $x'' + 4x = \cos t, \qquad x(0) = 1, \qquad x'(0) = 2.$

4.2. $x'' - 4x = t^2 + 6t - 2, \qquad x(0) = 0, \qquad x'(0) = 0.$

[16]Автор – И.П.Рязанцева. Для образца решения см. пример 4.21 в Части 4, т.1.

4.3. $x''-7x'+6x = 5e^t$, $x(0) = 1$, $x'(0) = -1$.

4.4. $x''+2x'+5x = 3\cos t$, $x(0) = 0$, $x'(0) = 0$.

4.5. $x''+4x'+4x = 5e^{-2t}$, $x(0) = -1$, $x'(0) = 1$.

4.6. $x''+x = t$, $x(0) = 0$, $x'(0) = 2$.

4.7. $x''+6x' = t^2 - 5t - 1$, $x(0) = 2$, $x'(0) = -2$.

4.8. $x''-x = te^t$, $x(0) = 1$, $x'(0) = -1$.

4.9. $x''-2x'+2x = te^t$, $x(0) = 1$, $x'(0) = 1$.

4.10. $x''-3x'+2x = te^{2t}$, $x(0) = -2$, $x'(0) = -2$.

4.11. $x''-2x'+5x = \cos 3t$, $x(0) = 2$, $x'(0) = 2$.

4.12. $x''+6x'+9x = 5e^{-3t}$, $x(0) = -1$, $x'(0) = -2$.

4.13. $x''+4x'-12x = 5\sin 3t$, $x(0) = 1$, $x'(0) = 1$.

4.14. $x''-2x' = t^2 - 3t$, $x(0) = 3$, $x'(0) = 3$.

4.15. $x''+2x' = te^{-2t}$, $x(0) = 1$, $x'(0) = -1$.

4.16. $x''-4x'+4x = 5te^{-2t}$, $x(0) = 0$, $x'(0) = 0$.

4.17. $x''+2x'+x = 5e^{-t}$, $x(0) = 1$, $x'(0) = 3$.

4.18. $x''+2x'+5x = te^{-t}$, $x(0) = 0$, $x'(0) = 0$.

4.19. $x''-2x'+5x = \sin 2t$, $x(0) = 0$, $x'(0) = 1$.

4.20. $x''-4x' = t^2 - 3$, $x(0) = 1$, $x'(0) = 0$.

4.21. $x''+4x' = 3e^t$, $x(0) = 0$, $x'(0) = 0$.

4.22. $x''-6x'+9x = \cos 3t$, $x(0) = 0$, $x'(0) = 1$.

4.23. $x''-7x'-8x = t^2 + 3t$, $x(0) = 1$, $x'(0) = 1$.

4.24. $x''+7x'-8x = 2e^{-t}$, $x(0) = -1$, $x'(0) = 1$.

4.25. $x''-5x'+4x = e^{-t}$, $x(0) = 2$, $x'(0) = 0$.

4.26. $x''+5x'+4x = e^{-t}$, $x(0) = 0$, $x'(0) = 2$.

4.27. $x''-9x'+20x = t^2 - 5$, $x(0) = 1$, $x'(0) = 0$.

4.28. $x''-x'-20x = t^2 + 5t$, $x(0) = 0$, $x'(0) = -1$.

4.29. $x''-9x = \cos 3t$, $x(0) = 1$, $x'(0) = -1$.

4.30. $x''+9x = \sin 2t$, $x(0) = 0$, $x'(0) = 2$.

Задача 5[17]. Методом операционного исчисления найти частное решение заданной системы дифференциальных уравнений.

5.1. $\begin{cases} x' = 8x-6y, \\ y' = 7x-5y, \end{cases}$ $x(0) = 0$, $y(0) = 1$.

5.2. $\begin{cases} x' = -7x+y, \\ y' = -2x-5y, \end{cases}$ $x(0) = 1$, $y(0) = 0$.

5.3. $\begin{cases} x' = x, \\ y' = 2x+2y, \end{cases}$ $x(0) = -1$, $y(0) = 0$.

5.4. $\begin{cases} x' = x+y, \\ y' = x-y, \end{cases}$ $x(0) = 1$, $y(0) = 1$.

5.5. $\begin{cases} x' = 4x+4y, \\ y' = 6x+2y, \end{cases}$ $x(0) = 2$, $y(0) = 0$.

5.6. $\begin{cases} x' = 3x+8y, \\ y' = x+y, \end{cases}$ $x(0) = -3$, $y(0) = 1$.

5.7. $\begin{cases} x' = -5x-2y, \\ y' = -4x-3y, \end{cases}$ $x(0) = 4$, $y(0) = 2$.

[17]Автор – И.П.Рязанцева. Для образца решения см. пример 4.22 в Части 4, т.1.

5.8. $\begin{cases} x' = 6x-8y, \\ y' = 3x-5y, \end{cases}$ $x(0) = -2,$ $y(0) = 1.$

5.9. $\begin{cases} x' = 3x+2y, \\ y' = -2x+8y, \end{cases}$ $x(0) = 2,$ $y(0) = -1.$

5.10. $\begin{cases} x' = -4x-4y, \\ y' = -6x-2y, \end{cases}$ $x(0) = -1,$ $y(0) = 4.$

5.11. $\begin{cases} x' = -x-7y, \\ y' = -5x-3y, \end{cases}$ $x(0) = 1,$ $y(0) = 3.$

5.12. $\begin{cases} x' = -5x-3y, \\ y' = -8x-3y, \end{cases}$ $x(0) = 0,$ $y(0) = 2.$

5.13. $\begin{cases} x' = -7x+4y, \\ y' = 5x-8y, \end{cases}$ $x(0) = 0,$ $y(0) = 5.$

5.14. $\begin{cases} x' = 2x+6y, \\ y' = 4x+4y, \end{cases}$ $x(0) = 2,$ $y(0) = 2.$

5.15. $\begin{cases} x' = x+y, \\ y' = 8x+3y, \end{cases}$ $x(0) = 1,$ $y(0) = 1.$

5.16. $\begin{cases} x' = 3x+5y, \\ y' = x-y, \end{cases}$ $x(0) = 3,$ $y(0) = 1.$

5.17. $\begin{cases} x' = -3x-4y, \\ y' = -2x-5y, \end{cases}$ $x(0) = 4,$ $y(0) = 2.$

5.18. $\begin{cases} x' = -5x+3y, \\ y' = -8x+6y, \end{cases}$ $x(0) = -4,$ $y(0) = 1.$

5.19. $\begin{cases} x' = 8x-2y, \\ y' = 2x+3y, \end{cases}$ $x(0) = 0,$ $y(0) = 4.$

5.20. $\begin{cases} x' = -2x-6y, \\ y' = -4x-4y, \end{cases}$ $x(0) = -1,$ $y(0) = -2.$

5.21. $\begin{cases} x' = -3x-5y, \\ y' = -7x-y, \end{cases}$ $x(0) = 1,$ $y(0) = 2.$

5.22. $\begin{cases} x' = -3x-8y, \\ y' = -3x-5y, \end{cases}$ $x(0) = -3,$ $y(0) = 3.$

5.23. $\begin{cases} x' = -5x+y, \\ y' = -2x-7y, \end{cases}$ $x(0) = -2,$ $y(0) = 4.$

5.24. $\begin{cases} x' = 4x+4y, \\ y' = -6x-7y, \end{cases}$ $x(0) = -1,$ $y(0) = 3.$

5.25. $\begin{cases} x' = -7x+4y, \\ y' = -6x+4y, \end{cases}$ $x(0) = 1,$ $y(0) = -2.$

5.26. $\begin{cases} x' = 5x, \\ y' = 2x-8y, \end{cases}$ $x(0) = 3,$ $y(0) = -1.$

5.27. $\begin{cases} x' = -8x, \\ y' = 4x+5y, \end{cases}$ $x(0) = 2,$ $y(0) = 1.$

5.28. $\begin{cases} x' = 5x-5y, \\ y' = 6x-8y, \end{cases}$ $x(0) = 0,$ $y(0) = 5.$

5.29. $\begin{cases} x' = -8x-5y, \\ y' = 6x+5y, \end{cases}$ $x(0) = -1,$ $y(0) = -3.$

5.30.
$$\begin{cases} x' = 5x - 11y, \\ y' = 2x - 8y, \end{cases} \qquad x(0) = 1, \qquad y(0) = -3.$$

Задача 6[18]. Найти спектральную функцию, построить амплитудный и фазовый спектры функции $f(t)$.

6.1. $f(t) = \begin{cases} -2, & t \in (-2;0), \\ 2, & t \in (0;2), \\ 0, & t \notin (-2;2). \end{cases}$ 6.2. $f(t) = \begin{cases} 2, & |t| < 4, \\ 0, & |t| > 4. \end{cases}$

6.3. $f(t) = \begin{cases} 3, & t \in (0;4), \\ 0, & t \notin (0;4). \end{cases}$ 6.4. $f(t) = \begin{cases} |t|, & |t| < 1, \\ 0, & |t| > 1. \end{cases}$

6.5. $f(t) = \begin{cases} e^{-2t}, & t > 0, \\ 0, & t < 0. \end{cases}$ 6.6. $f(t) = \begin{cases} 0, & t > 0, \\ e^{3t}, & t < 0. \end{cases}$

6.7. $f(t) = \begin{cases} 1, & |t| < 1, \\ 2, & 1 < |t| < 2, \\ 0, & |t| > 2. \end{cases}$ 6.8. $f(t) = \begin{cases} \cos 3t, & |t| < \pi/3, \\ 0, & |t| > \pi/3. \end{cases}$

6.9. $f(t) = \begin{cases} \sin 4t, & |t| < \pi/4, \\ 0, & |t| > \pi/4. \end{cases}$ 6.10. $f(t) = \begin{cases} 1+t, & t \in (-1;0), \\ 1-t, & t \in (0;1), \\ 0, & t \notin (-1;1). \end{cases}$

6.11. $f(t) = \begin{cases} 2, & t \in (-3;0), \\ 0, & t \notin (-3;0). \end{cases}$ 6.12. $f(t) = \begin{cases} 2, & t \in (2;4), \\ 0, & t \notin (2;4). \end{cases}$

6.13. $f(t) = \begin{cases} 1 - \frac{|t|}{3}, & |t| < 3, \\ 0, & |t| > 3. \end{cases}$ 6.14. $f(t) = \begin{cases} 2, & |t| \in (1;2), \\ 0, & |t| < 1, |t| > 2. \end{cases}$

6.15. $f(t) = \begin{cases} 2, & t \in (1;2), \\ -2, & t \in (-2;-1), \\ 0, & |t| < 1, |t| > 2. \end{cases}$ 6.16. $f(t) = \begin{cases} 2 - 3|t|, & |t| < 1/3, \\ 0, & |t| > 1/3. \end{cases}$

6.17. $f(t) = \begin{cases} 3, & t \in (-2;0), \\ -3, & t \in (0;2), \\ 0, & t \notin (-2;2). \end{cases}$ 6.18. $f(t) = \begin{cases} -1, & 2 < |t| < 3, \\ 0, & |t| < 2, |t| > 3. \end{cases}$

6.19. $f(t) = \begin{cases} 4, & t \in (-4;0), \\ 0, & t \notin (-4;0). \end{cases}$ 6.20. $f(t) = \begin{cases} 2, & t \in (-4;-2), \\ 0, & t \notin (-4;-2). \end{cases}$

[18]Автор – И.П.Рязанцева. Для образца решения см. пример 5.2 в Части 4, т.1.

6.21. $f(t) = \begin{cases} 2 + 3t, & t \in (-\frac{1}{3};0), \\ 2 - 3t, & t \in (0;\frac{1}{3}), \\ 0, & |t| > \frac{1}{3}. \end{cases}$ 6.22. $f(t) = \begin{cases} -2|t|, & |t| < 1/2, \\ 0, & |t| > 1/2. \end{cases}$

6.23. $f(t) = \begin{cases} 2, & t \in (-1;0), \\ -1, & t \in (0;1), \\ 0, & |t| > 1. \end{cases}$ 6.24. $f(t) = \begin{cases} \cos 2t, & t \in (0;\pi/2), \\ 0, & t \notin (0;\pi/2). \end{cases}$

6.25. $f(t) = \begin{cases} \sin 3t, & t \in (-\pi/3;0), \\ 0, & t \notin (-\pi/3;0). \end{cases}$ 6.26. $f(t) = e^{-|t|}$.

6.27. $f(t) = \begin{cases} 1 + \cos 2t, & |t| < \pi/2, \\ 0, & |t| > \pi/2. \end{cases}$ 6.28. $f(t) = \begin{cases} -3, & t \in (-2;0), \\ 2, & t \in (0;2), \\ 0, & |t| > 2. \end{cases}$

6.29. $f(t) = \begin{cases} \sin 4t, & t \in (0;\pi/4), \\ 0, & t \notin (0;\pi/4). \end{cases}$ 6.30. $f(t) = \begin{cases} \cos 2t, & t \in (-\frac{\pi}{2};0), \\ 0, & t \notin (-\frac{\pi}{2};0). \end{cases}$